A Solution to the Shortcomings of American Geometry Education

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Chapter 1

The Current State of High School Geometry

"I don't see how it's doing society any good to have it's members walking around with vague memories of algebraic formulas and geometric diagrams, and clear memories of hating them."

— Paul Lockhart, *A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form*[1]

The year is 530 BCE, and your name is Pythagoras. You spend your days traversing your hometown island, Samos. After greeting your local acquaintances and purchasing a small loaf of bread and a dollop of jam with a few drachmas, you set off for a shaded hill near the beach. The quiet frothing of the waves, and the salty breeze helps you think. Numbers are something that captivate you, and you see their mathematical beauty in the land, plants, and even yourself. As the sun totters its way to its perch in the sky, you feel too tired to think, as if the sun had been quietly singing a lullaby while you were entranced in thought. You slowly shut your eyes, bringing your ongoing thoughts into the land of dreams.

When you open your eyes, you remember you've dozed off in geometry class, and this time Pythagoras is but a name in a textbook. His theorem is something you need to use if you want a good grade. Rather than enjoying math in the natural world while the ocean serenades you, the only noises around you are the monotonous bee-like drones of your teacher and the quiet tittering of classmates. As you look down at your worksheet, you mindlessly apply a succession of theorems to show that two triangles are congruent. You doubt the usefulness of anything you're doing. Your only reprise is to watch the classroom clock with suspense as it hangs in the air, stagnantly facing you with a mocking sneer. What a bore!

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Unfortunately, this experience has become increasingly common for mathematics education in America. In fact, mathematics has been noted to be the least popular subject in American education[2]. Throughout an average American student's mathematical career, a student will tediously learn a non-trivial amount of acronyms, theorems, and formulae. The parochial nature of this education prepares masses of students for assignments and exams, but it starves them of individualistic thinking processes necessary for higher level education. By emphasizing memorizing *what* formulae to use rather than *how* formulae work and *why* they should be used, students are subjugated to mindless thinking patterns.

The most concerning offender of this type of mathematical "education" is geometry. This class is likely the first time that students are exposed to proofs, a pure form of math that requires creativity and logic to reason through abstract problems; however, the American education system has disfigured the topic to disincentivize creative thought in preference for rote memorization. It's no wonder that amidst this mass of "knowledge" many students lose their love for mathematics or worse, may believe they are "gifted" mathematicians despite only boasting a strong memory. The preparation for these students is lackluster and sets them up for failure. Only when these gifted math students reach college, where instruction is more open-ended, do they realize that their creative thinking is atrophied and may fall behind.

Even when students try to love geometry and find the topic interesting, systemic issues impede their goals. Students are excited to learn more, often outpacing their teachers, but unfortunately, nearly a third of licensed high-school math teachers have not had any collegiate experience with math and may have a difficult time expressing higher level mathematics[3]. Through no fault of their own, teachers who try their best just might not know mathematics well enough to teach it. Instead, they have to rely on rigid, state-provided curricula that fails to capture the learning styles of all students in a classroom. What follows is a teacher that dominates the classroom environment, limiting bidirectional communication, an integral part of mathematical thinking. This creates a self-perpetuating cycle of one-sided instructors and unmotivated students.

Finally, the geometry problems that students encounter are so disconnected from reality that many students truly believe that what they are learning has no applicable benefits. Whether it's basic algebra or geometry, a common question of "why would I need to know this?" hangs in the air of the classroom. While it might seem counter-intuitive, in a geometry setting, classes should not emphasize the importance of learning geometry. Geometrical theorems should be a byproduct of understanding how to reason proof-based problems, and *reasoning* is a realistic skill that students can take away. By acknowledging the abstractness of the problems they are trying to solve, students may be more willing to learn the practical skills to tackle these unrealistic problems.

Altogether, I believe that the American geometry education can be characterized as a parochial, one-sided, unrealistic, and rote (POUR) system. For over two centuries, this system has left little room for individualistic growth and disincentivizes creative reasoning, an essential skill for any field. The POUR system continues to leave chronic consequences in society. Possibly the most concerning is that a person's childhood is the most impressionable part of their life. By indoctrinating young students with this close-minded thinking process, we can not possibly expect them to flourish and collaborate freely in open environments. In *Dumbing Us Down*, David Albert notes that from the 1970s to the 2000s, the largest growing jobs were Wal-Mart clerks, MacDonald's cooks, Burger King cooks, and elementary school teachers[3]. Education is a commodity, and Americans are actively investing into this public education and effectively reinforcing this cycle. Of course, some students manage to thrive by making adjustments to their thinking. This capitulation to the system should not be normalized. Instead, we need to reevaluate the priorities of our education system; do we want to create machines or free-thinkers?

Chapter 2

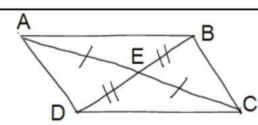
A Solution

"The culture we have does not make people feel good about themselves. And you have to be strong enough to say if the culture doesn't work, don't buy it"

— Morrie Schwartz, *Tuesdays with Morrie*[4]

Consider this proof:

Proof #5

Given: $\overline{AE} \cong \overline{EC}$ and $\overline{DE} \cong \overline{EB}$ Prove: ABCD is a p'gram	
Statements	**Reasons**
1. $\overline{AE} \cong \overline{EC}$ and $\overline{DE} \cong \overline{EB}$	1. Given
2. $\angle BEC \cong \angle AED$	2. Vertical $\angle$'s $\cong$
3. $\Delta BEC \cong \Delta AED$	3. SAS congruence postulate
4. $\angle CBE \cong \angle ADE$	4. CPCTC
5. $\overline{BC} \parallel \overline{AD}$	5. Alt int $\angle$'s $\cong$ $\rightarrow$ lines $\parallel$
6. $\angle AEB \cong \angle CED$	6. Vertical $\angle$'s $\cong$
7. $\Delta AEB \cong \Delta CED$	7. SAS congruence postulate
8. $\angle DCE \cong \angle BAE$	8. CPCTC
9. $\overline{AB} \parallel \overline{DC}$	9. Alt int $\angle$'s $\cong$ $\rightarrow$ lines $\parallel$
10. ABCD is a p'gram	10. Defn of a parallelogram

Did you actually read it? Regardless, what you see is an eyesore, and to many students seeing geometry for the first time, they may feel overwhelmed. Geometry and other proof-based educations should be free, not constrained between a two-column form that may cost students points from their grade if they make a minor error. We need an undogmatic education system that tests reasoning rather than adherence to rules.

I propose a solution of introducing a hybrid-flipped classroom. In a flipped classroom, student and teacher collaboration is the focus of the class, rather than lecture. It's important that students still receive some instruction. Transitioning straight into a flipped classroom, where class time is not spent on lecture, can be too unfamiliar, especially for novice mathematicians. Students will alternate between lecture-based classes and collaborative problem-solving classes. This creates an environment with novelty and familiarity. By exposing them to a more liberal learning scheme, they can develop their thought processes and enjoy math as a creative science.

The discussion-based learning between students and teachers helps to gamify and create a friendly learning environment. Instead of a one-sided lecture, there will be a multi-sided conversations that help germinate new ideas. This way, students can be instructed on both basic fundamental theorems and be encouraged to collaborate with those around them to solve geometry problems. The shortcomings of some students can be supplemented by sudden inspirations of other students, leading to a greater mathematical maturity. Furthermore, more students may feel emboldened to speak up in situations when they would have otherwise remained silent. In a study by Rowe, working in groups and allowing students time to think after questions has a multitude of benefits including: increased speculative thinking, wider range of participation, and greater student participation[5]. This is in contradiction to a large classroom dominated by a single figure, where it may be difficult to find time to interrupt the lesson for questions.

Problems in this class style are meant to be difficult, span multiple days, and build off of each other. No longer will problems be independent. In this sense, they will be realistic. Students will learn to construct lemmas and plan to tackle larger problems. This classroom style facilitates tearing down parochial and unrealistic learning habits ingrained in students over the years. In particular, studies have shown that metacognitive aspects of mathematical learning are much more important than calculating solutions; more precisely, it is imperative for students to understand how they think. During the flipped classroom portion, having a teacher around to elucidate conceptual ideas and guide them in discussion raises students' mathematical maturity and coerces them to think of various plans to tackle a problem[6]. Through this protocol, the teacher rejects the "sage on the stage" persona and adopts the position as a "guide on the side."

Altogether, this flipped classroom style helps combat the downsides of the POUR education system in an approachable manner. The reasons for this solution have been laid out, and it's only a matter of whether we wish to recognize the issues and work towards a solution. Truly, I believe that the earlier that this system is implemented the better for students in general.

Bibliography

1. Lockhart, Paul. A Mathematician's Lament. Bellevue Literary Press, 2009.
2. "Education: Least Popular Subject." Time, Time Inc., 18 June 1956, content.time.com/time/subscriber/article/0,33009,862253,00.html.
3. Gatto, John Taylor. Dumbing Us down: The Hidden Curriculum of Compulsory Schooling. New Society Publishers, 2017.
4. Albom, Mitch. Tuesdays with Morrie: An Old Man, a Young Man, and Life's Greatest Lesson. Crown, 2022.
5. Rowe, Mary Budd. "Wait time: Slowing down may be a way of speeding up!" Journal of Teacher Education, vol. 37, no. 1, Jan. 1986, pp. 43–50, https://doi.org/10.1177/002248718603700110.
6. Schoenfeld, Alan H. "What's all the fuss about metacognition?" Cognitive Science and Mathematics Education, 3 Apr. 2013, pp. 207–234, https://doi.org/10.4324/9780203062685-12.
7. Walker, Robert J. Twelve Characteristics of an Effective Teacher, Lulu.Com, Morrisville, North Carolina, 2020, pp. 61–68.